M E T U Department of Mathematics

Math 118 Calculus II Midterm II 27.07.2017 15:00		
Last Name : Name : Student No:	Signature : Duration : 100 minutes	Section :
5 QUESTIONS ON 4 PAGES		TOTAL 100 POINTS
1 | 2 | 3 | 4 | 5	SHOW YOUR WORK	

1. (12 pts) Is the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$ divergent, conditionally convergent or absolutely convergent?

$$\sum_{n=0}^{\infty} \left| \frac{(-1)^n}{\sqrt{n^2+1}} \right| = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$$ $\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}} = 1$, since $\sum \frac{1}{n}$ is divergent, from Limit Comparison Test, $\sum \frac{1}{\sqrt{n^2+1}}$ is divergent.

given series is NOT absolutely convergent.

Consider $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$ $b_n = \frac{1}{\sqrt{n^2+1}}$

i) $b_n \geq 0$, for all $n \geq 0$.

ii) $\{b_n\}$ decreasing

$f(x) = \frac{1}{\sqrt{x^2+1}}$, $f'(x) = \frac{-x}{(1+x^2)^{3/2}} < 0$

iii) $\lim_{n\to\infty} \{b_n\} = 0$.

Thus, given series is conditionally convergent.

2. (12 pts) Approximate the integral $\int_0^1 \frac{\sin(t)}{t} dt$ with an error less than 10^{-3}?

$$\frac{\sin t}{t} = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!\, t} \Rightarrow \int_0^1 \frac{\sin t}{t} dt = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!(2n+1)}.$$

$a_n = \frac{1}{(2n+1)!\,(2n+1)}$

(i) $a_n \geq 0$

(ii) $\{a_n\}$ decreasing

(iii) $\lim_{n\to\infty} a_n = 0$.

From error estimation theorem

$$|Error| \leq |a_{n+1}| < \frac{1}{1000} \Rightarrow \frac{1}{(2n+3)(2n+3)!} < \frac{1}{1000}$$

$\underline{n=2}$ $\frac{1}{7\cdot 7!} < \frac{1}{1000}$ True.

Result: $\int_0^1 \frac{\sin t}{t} dt \approx 1 - \frac{1}{3\cdot 3!} + \frac{1}{5\cdot 5!} \approx 0.946$.

3. (6+6+6+6 pts) Let $f(x) = \sum_{n=1}^{\infty} \frac{n^2(x+1)^{3n+2}}{8^n}$ be the given series.

a) Does the power series converge for $x = 1$?

$x = 1$ $\quad \sum_{n=1}^{\infty} 4n^2$, divergent from n-term test.

$\lim_{n\to\infty} 4n^2 = \infty \neq 0$.

b) Does the power series converge for $x = -3$?

$x = -3$ $\quad \sum_{n=1}^{\infty} 4n^2(-1)^n$, divergent from n-term test

$\lim_{n\to\infty} 4n^2(-1)^n = \mp\infty \neq 0$.

c) Determine the interval of the convergence of the given power series?

$$\lim_{n\to\infty} \left| \frac{(n+1)^2 (x+1)^{3n+5}}{8^{n+1}} \cdot \frac{8^n}{n^2(x+1)^{3n+2}} \right| = \lim_{n\to\infty} \frac{|x+1|^3}{8} \cdot \frac{(n+1)^2}{n^2}$$

$$= \frac{|x+1|^3}{8} < 1 \Rightarrow \quad |x+1| < 2 \Rightarrow \quad -3 < x < 1$$

From part (a) & (b) interval of convergence

$$\boxed{(-3, 1)}$$

d) Find $f^{(32)}(-1)$.

$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$ coefficient of $(x-c)^n = a_n = \frac{f^{(n)}(-1)}{n!}$

Need to find coefficient of x^{32}.

$3n+2 = 32 \Rightarrow n = 10$.

If $n=10$. (coefficient of x^{32}) $= a_{32} = \frac{100}{8^{10}} = \frac{f^{(32)}(-1)}{(32)!}$

$$f^{(32)}(-1) = \frac{100\,(32)!}{8^{10}}.$$

4. (6+6+6+6 pts) Determine whether the following series are convergent or divergent.

a) $\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(-2)^{2n}}{n^n}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{2^{2n}}{n^n}} = \lim_{n\to\infty} \frac{4}{n} = 0 < 1$$

From root test, given series is absolutely convergent so, it is convergent

b) $\sum_{n=1}^{\infty} \frac{3^n}{n^5+3^n}$

$$\lim_{n\to\infty} \frac{3^n}{n^5+3^n} = \lim_{n\to\infty} \frac{\cancel{3^n}}{\cancel{3^n}\left(\frac{n^5}{3^n}+1\right)} = 1 \neq 0.$$

$\frac{n^5}{3^n} \to 0.$

From n^{th} term test, given series is divergent.

c) $\sum_{n=1}^{\infty} \frac{(\ln n)^5}{n}$

$a_n = \frac{(\ln n)^5}{n} > 0, \ n \geq 1. \quad b_n = \frac{1}{n}.$

$$\lim_{n\to\infty} \frac{\frac{(\ln n)^5}{n}}{\frac{1}{n}} = \infty, \text{ i.e. } \frac{1}{n} < \frac{(\ln n)^5}{n},$$

from Limit Comparison test, given series is divergent.

(or $\frac{(\ln n)^5}{n} > \frac{1}{n}$ & $\sum \frac{1}{n}$ div)

d) $\sum_{n=0}^{\infty} \frac{2^n+4^n}{5^n}$

$$\lim_{n\to\infty} \frac{\frac{2^n+4^n}{5^n}}{\left(\frac{4}{5}\right)^n} = \lim_{n\to\infty} \frac{2^n+4^n}{4^n} = 1.$$

Since $\sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n$ geometric series, $|r| = \left|\frac{4}{5}\right| < 1$, is convergent, given series is convergent.

5.(7+7+7+7 pts) Given two lines

$$L_1 : x = 1 + 2t,\ y = -t,\ z = 2 - 2t, \qquad L_2 : x = 1 - s,\ y = 1 + s,\ z = 1 + s$$

a) Decide whether the lines L_1 and L_2 are intersecting. Show your work.

$$\left.\begin{array}{l} 1+2t = 1-s \\ -t = 1+s \end{array}\right\} \Rightarrow s = -2t$$

$$-t = 1-2t \Rightarrow t = 1,\ s = -2$$

$2-2t = 1+s$ plug in $t=1$ $s=-2$ into

$$2-2t = 1+s$$
$$0 \neq -1$$

L_1 and L_2 do not intersect.

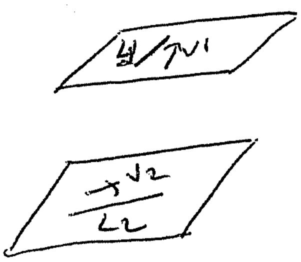
b) Decide if L_1 and L_2 are skew. Show your work.

Let $\vec{v_1}$ be the direction vector of L_1 and $\vec{v_2}$ be the direction vector of L_2.

$\vec{v_1} = \langle 2,-1,-2 \rangle$ $\vec{v_2} = \langle -1,1,1 \rangle$; $\vec{v_1} \neq k\vec{v_2}$, $k \in \mathbb{R}$ not parallel.

From part (a); skew lines

c) Find the equation of a plane containing L_1 and parallel to L_2.



$\vec{n} = \vec{v_1} \times \vec{v_2}$ (common normal to both planes)

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & -2 \\ -1 & 1 & 1 \end{vmatrix} = \vec{i} + \vec{k}$$

plane equation:

$$\langle 1,0,1 \rangle \cdot \langle x-1, y, z-2 \rangle = 0.$$

$$\boxed{x+z=3}$$

d) Find the distance between L_1 and L_2.

Point from L_2: $(1,1,1)$

$$d = \frac{|1+1-3|}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Declaration of Honesty: By signing below, I pledge that I will write this examination as my own work and without the assistance of others or the usage of unauthorized material or information. I understand that possession of any kind of electronic device during the exam is prohibited. I also understand that not obeying the rules of the examination will result in immediate cancellation and disciplinary procedures.

Signature : ....